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A Numerical Solution for the Nonlinear Diffusion Equation of the Electromagnetic Field in Ferromagnetic Materials

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In this paper the diffusion equation of the electromagnetic field in ferromagnetic materials is studied. The main difficulty in this equation lies in the nonlinear magnetic characteristic, that results in a nonlinear diffusion equation. To select a finite difference scheme to solve this nonlinear diffusion equation, a comparative analysis of the main difference schemes is made. Given the difficulties introduced by the nonlinear magnetic relationship we suggest an ordered comparative study. © 1986 Academic Press, Inc.

1. INTRODUCTION

The diffusion equation of an electromagnetic field is a partial differential equation of parabolic type. If we deal with ferromagnetic media, an additional complexity arises due to the nonlinear B/H relationship. The differential equation is then of the form

$$u_{xx} + \beta(u)\, u_t = 0 \tag{1}$$

where β is a function of the dependent variable but not of its derivatives. The equation is thus called quasi-linear [1].

The state-of-the-art in this field is still very fragmentary [2]. Given the incomplete theoretical knowledge of these matters, only numerical experimentation can ultimately decide what is the best finite difference scheme for a parabolic problem. This experimentation should be carried out with comparative studies, but as Lim and Hammond [3] point out, these studies are unfortunately rarely shown in the literature, resulting in the absence of a practical comparison.

The problem of selecting a finite-difference method to solve the electromagnetic field diffusion equation is treated in this paper. Given that the behavior of the equation is strongly dependent on the type of B/H relationship, we decided on a comparative study of the difference schemes for each type of relationship. As a reference to make the comparison, we first considered a linear magnetic characteristic, which allowed us to solve the diffusion equation with linear techniques and establish the reference for the comparative study.

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2. The Diffusion Equation and the B/H Relationship

The behavior of an electromagnetic field is basically determined by the diffusion equation

$$\nabla^2 \mathbf{H} = \sigma \frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

where **H** is the magnetic field and **B** the flux density.

Considering the 1-dimensional equation for the Z component of the field inside a conductor plate of uniform conductivity gives (Fig. 1)

$$\frac{\partial^2 H}{\partial x^2} = \sigma \frac{\partial B}{\partial H} \frac{\partial H}{\partial t} \tag{3}$$

with the boundary conditions

$$H(x, 0) = 0 \qquad -d \leqslant x \leqslant d \tag{4}$$

$$H(\pm d, t) = H_0 \sin \omega t \qquad 0 \leqslant t \tag{5}$$

where H_0 is the surface field amplitude. Only half the lamination need be considered as H is symmetrical about the center plane and hence

$$\left.\frac{\partial H}{\partial x}\right|_{x=0} = 0 \qquad 0 \leqslant t \tag{6}$$

can be used as an alternative boundary condition.

The diffusion equation is nonlinear through the term related to the B/H relationship. For convenience we will write

$$\beta = \sigma \frac{\partial B}{\partial H} \tag{7}$$

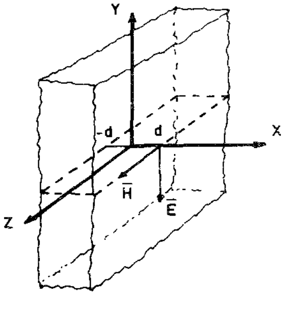

FIG. 1. Conductor plate.

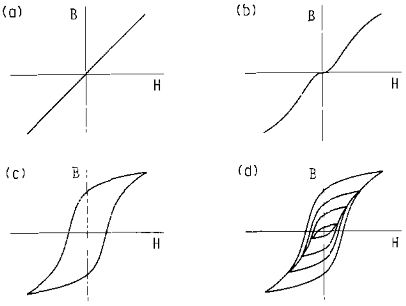

FIG. 2. Magnetic relationships: (a) linear curve; (b) nonlinear monovalued curve; (c) nonlinear bivalued curve; (d) family of nonlinear bivalued curves.

and throughout the study, this will be a constant or a function of H, according to the magnetic characteristic being considered.

To obtain a solution for the diffusion equation one needs to know the relationship of magnetic field strength H to flux density B. Depending on the type of material and magnetic behavior simplifications, the magnetic characteristic can be approximated by a straight line, a monovalued curve, a bivalued curve or a family of bivalued curves, each loop corresponding to a depth inside the material (Fig. 2). For thick ferromagnetic samples under *ac* magnetization—our main objective—the B/H relationship would be a family of bivalued curves, constructed in such a manner that in the steady state each curve applies to a specific depth.

The numerical behavior of the finite-difference schemes in nonlinear equations is not well known, and the situation is still worse if we are dealing with a nonlinearity as complex as that in Fig. 2d. With the objective of finding the most suitable difference method, we suggest an ordered comparative study of the difference schemes. Beginning with the linear B/H relationship, which has a well known solution that allows us to establish a reference case, we then consider a nonlinear monovalued curve; then we introduce the bivalued function; and finally the depth variation is introduced. For every additional difficulty, the numerical behavior of the schemes is studied, rejecting unstable ones or those with serious difficulties. MKS units are used throughout the study.

3. SOLUTION OF THE DIFFUSION EQUATION: DIFFERENCE SCHEMES

When a finite difference method is used, the continuous variable is replaced by a discrete variable that only possesses a value at the nodes in a space-time grid. If the plate surface is taken as the origin, the coordinates are

$$x = (i-1)\,\Delta x \qquad i = 1, 2, 3, \ldots, M \tag{8}$$

$$t = (k-1)\,\Delta t \qquad k = 1, 2, 3, \ldots, N \tag{9}$$

where the mesh length Δx is the distance between adjacent nodes in space and Δt is the time interval between successive values of H_i appearing at the space node i. It is helpful to think of the solution as marching forward in time, each step moveing Δt, generating the electromagnetic transient as it progresses.

An important question is the choice of the space-line grid. In this paper, the final space-time grid considered for each case, is that which gives values which cannot be significantly improved by making the grid finer.

In this study, the significant difference schemes wil be considered: the simple explicit [2, 4], the simple implicit [2, 4], the Dufort–Frankel [5] and the Crank–Nicolson [6] schemes. If the intermedial variables

$$\rho = \beta(\Delta x)^2/\Delta t \tag{10}$$

$$r = 1/\rho \tag{11}$$

$$\alpha = 2r/(2r+1) \tag{12}$$

are defined, the finite-difference analog of the diffusion equation, for each scheme, are respectively

$$H_{i,k+1} = rH_{i-1,k} + (1-2r)\, H_{i,k} + rH_{i+1,k} \tag{13}$$

$$H_{i-1,k+1} - (2r+\rho)\, H_{i,k+1} + H_{i+1,k+1} = -\rho H_{i,k} \tag{14}$$

$$H_{i,k+1} = \alpha H_{i-1,k} + (1-2\alpha)\, H_{i,k-1} + \alpha H_{i+1,k} \tag{15}$$

$$H_{i-1,k+1} - (2-2\rho)\, H_{i,k+1} + H_{i+1,k+1} = -H_{i-1,k} + (2-2\rho)\, H_{i,k} - H_{i+1,k}. \tag{16}$$

Equations (13) and (15) are explicit, as $H_{i,k+1}$ is derived solely from in the previous time rows. The other two equations (14) and (16) are implicit as there are three new values of the dependent variable which have to be solved simultaneously with the values that appear in the finite-difference equations associated with the other nodes. The coefficient matrix of the set of equations is tridiagonal and the set can be readily solved by gaussian elimination [7].

The numerical solution is a step-by-step process and gradually moves through the transient to the steady state field distribution, which is the objective. If N is the number of time steps corresponding to a half cycle of the surface field, then the solution reaches the steady state when.

$$H_{i,k} = -H_{i,k-N}. \tag{17}$$

To check this,

$$Q = H_{M,k} - H_{M,k-N} \tag{18}$$

is examined at the end of a half cycle. The center node M is chosen for improved accuracy. Thus, the steady state is reached when

$$Q \leqslant Q_{\text{ref}} \tag{19}$$

where Q_{ref} is specified as a fraction of the amplitude of the center plane field.

It is sometimes useful to attempt to accelerate the transient, in such a way that Eq. (17) be fulfilled. Therefore, we require a correction factor K_i, such that

$$(H_{i,k}-K_i)+(H_{i,k-N}-K_i)=0. \tag{20}$$

Then this correction factor is given by

$$K_i=\tfrac{1}{2}(H_{i,k}+H_{i,k-N}) \tag{21}$$

and the corrected values of $H_{i,k}$ which we call $H'_{i,k}$ are

$$H'_{i,k}=H_{i,k}-K_i=\tfrac{1}{2}(H_{i,k}-H_{i,k-N}). \tag{22}$$

If we are working near the "knee" of the magnetization characteristic, application of (22) once every half-cycle can reduce the computation required to reach the steady state solution by more than half [8], otherwise, the improvement may be negligible.

4. Comparative Analysis for the Linear Problem

As a first step towards the comparative analysis of the difference schemes considered, the stability problem was studied. The simple implicit, Dufort–Frankel and Crank–Nicolson schemes are stable for all values of Δx and Δt [1, 2, 4, 7]. However, this is not the case for the simple explicit scheme, which suffers from the severe restriction [2, 4]

$$r\leqslant\tfrac{1}{2} \tag{23}$$

which is necessary to ensure stability. The coefficient β (7) will never be less than two for the conductive materials of interest, so that in the worst case the time interval is so small that the scheme is computationally inefficient. For this reason, we rejected this scheme and only considered the other three schemes.

The question of convergency of numerical methods is difficult to show and so the looser concept of compatibility is often used [7, 8, 9]. This is associated with the truncation error which is the difference between the partial differential and the finite-difference equations rather than that of their solutions. If L is the partial differential operator and L_f the finite difference operator the truncation error T is defined as follows

$$T=L_fH-LH. \tag{24}$$

If T tends to zero as Δx and Δt tend to zero, the finite-difference scheme is compatible. For linear initial value problems, stability and compatibility imply convergence [8, 10, 11].

The truncation errors of the schemes are given by

$$\text{Simple explicit} \qquad O(\Delta t+(\Delta x)^2) \tag{25}$$

$$\text{Simple implicit} \qquad O(\Delta t+(\Delta x)^2) \tag{26}$$

$$\text{Dufort–Frankel} \qquad O\left((\Delta t)^2+(\Delta x)^2+\left(\frac{\Delta t}{\Delta x}\right)^2\right) \tag{27}$$

$$\text{Crank–Nicolson} \qquad O((\Delta t)^2+(\Delta x)^2) \tag{28}$$

obtained from the neglected terms of the Taylor series. The truncation error of the Dufort–Frankel scheme has an additional term which implies that the ratio $\Delta t/\Delta x$ must tend to zero as Δx and Δt separately tend to zero. Otherwise we would be solving the hyperbolic equation

$$\frac{\partial^2 H}{\partial x^2}-\beta\frac{\partial H}{\partial t}-\left(\frac{\Delta t}{\Delta x}\right)^2\frac{\partial^2 H}{\partial t^2}=0. \tag{29}$$

In practice, if

$$\left(\frac{\Delta t}{\Delta x}\right)^2 \ll \beta \tag{30}$$

the Dufort–Frankel scheme is compatible with the diffusion equation [8].

After considering the above, we can say that

(a) The truncation errors of the simple explicit and implicit schemes are of the same order, but due to the stability restriction of the former its disadvantage with respect to the latter is evident.

(b) With a given Δx and Δt and as long as Eq. (30) is satisfied, the Dufort–Frankel and Crank–Nicolson schemes are more accurate than the simple implicit scheme.

(c) The Crank–Nicolson scheme has a lower truncation error than the Dufort–Frankel scheme but, as it is an implicit method, it is also slower. Thus, for certain applications, the Dufort–Frankel can be more convenient.

To summarize for linear problems, if a given accuracy is desired, the time interval for the Dufort–Frankel scheme is greater than for the simple implicit scheme, while that for the Crank–Nicolson scheme can be superior to both. The time interval for the simple explicit scheme is the lowest, and, if Eq. (23) is not satisfied, the scheme is unstable.

Taking into account the error relationships, the superiority of the Dufort–Frankel and Crank–Nicolson schemes is evident. However, choosing between them is not obvious, because although the Crank–Nicolson has no restriction with respect to the interval ratio, it is an implicit scheme, whereas the Dufort–Frankel scheme exhibits all the advantages of an explicit method.

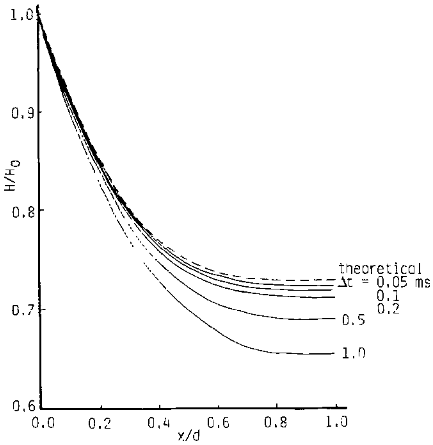

FIG. 3. Calculated amplitudes of H as a function of the depth in a linear material, using the simple implicit scheme.

As a graphical verification of the analysis, and with the objective of studying the mesh discretization effect, we have represented the calculated solutions. In Figs. 3, 4, and 5 the computed amplitudes for several time intervals is shown, together with the theoretical solution. The problem has been solved for a hypothetical, magnetically linear sheet [3] with the following characteristics $\mu = \mu_0 = 4\pi 10^{-7} H\,\mathrm{m}^{-1}$, $\sigma = 5.8 \times 10^7 \Omega^{-1}\,\mathrm{m}^{-1}$, $f = 50$ Hz, $d = 10^{-2}$ m. For all computations, Δx was taken as 10^{-3} m. and the amplitude of the surface field was 100 A/m.

The convergence regularity of the implicit schemes can be observed with respect to the Dufort–Frankel scheme which, although it has a truncation error lower than

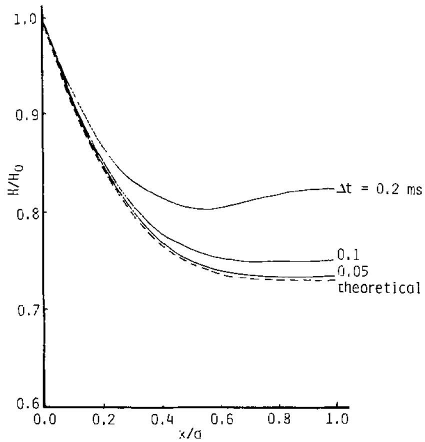

FIG. 4. Calculated amplitudes of H as a function of the depth in a linear material, using the Dufort–Frankel scheme.

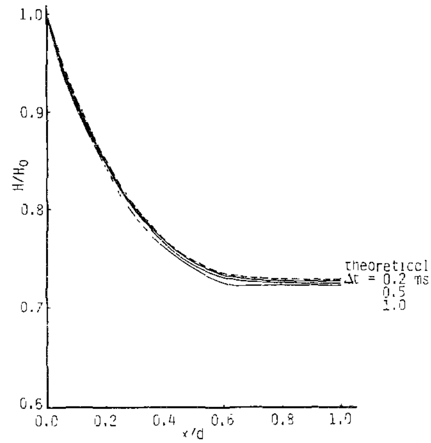

FIG. 5. Calculated amplitudes of H as a function of the depth in a linear material, using the Crank–Nicolson scheme.

that of the simple implicit scheme, converges slowly to the theoretical solution because of the condition (30). The Crank–Nicolson scheme stands out as it provides very satisfactory values with relatively large intervals. For the same intervals, the simple explicit scheme does not remain stable.

The conclusions obtained with respect to the behavior of the schemes for the linear problem act as a basis for comparison in the nonlinear problem. We will analyze the stability and convergence as a function of the mesh discretization, the effects of the nonlinearity on the computation process and the solutions.

5. The Nonlinear Problem

The fact that a partial differential equation is nonlinear, does not have to exclude a solution using linear techniques. Linear techniques can be used whenever the finite-difference representation can be expressed as a linear algebraic equation of its unknowns [7, 12]. For the diffusion equation, where the coefficient β is a function of the field H, the difference equation can be made if we derive values for β, which do not depend on $H_{i,k+1}$, on the new time row $(k+1)$.

This can be done if β is computed from values on the time row (k). However, in problems of this type the total error is given by the inherent error of the finite difference scheme and by the error obtained in the calculation of β. As the time step used in the implicit schemes is relatively large, the error of β can also be large if it is calculated on time row (k), especially when saturation is considered. In implicit schemes this error can be reduced by using a predictor-corrector method [8, 12], the difference equation thus remaining linear. This method consists of the calculation of the function on time row $(k+1)$ by evaluating β on time row (k).

The values thus calculated are "predicted" values and are relatively inaccurate. With these values, β can be estimated on time row $(k+1)$ and so the time row $(k+1)$ values can be recalculated, giving "corrected" values. This method is used for the simple implicit scheme. But for the Crank–Nicolson scheme, bearing in mind that the equation is centred on the intermediate node $(i, k+\frac{1}{2})$, we should estimate β at this point. The necessary value of H at node $(i, k+\frac{1}{2})$ can be obtained by using a half-time step Crank–Nicolson equation with β based upon node (i, k), so that

$$\rho_{i,k} = \beta_{i,k} \frac{(\Delta x)^2\, 2}{\Delta t}. \tag{31}$$

Having thus obtained $H_{i,k+1/2}$, the value of β can be calculated on the node $(i, k+\frac{1}{2})$. For the full Crank–Nicolson step from the kth to the $(k+1)$th time row the coefficient is

$$\rho_{i,k+1/2} = \beta_{i,k+1/2} \frac{(\Delta x)^2}{\Delta t}. \tag{32}$$

The use of a predictor-corrector method implies that for the simple implicit and the Crank–Nicolson schemes and for the same Δx and Δt, a nonlinear problem requires twice as much calculation as the linear one.

As for the Dufort–Frankel scheme, being an explicit method and because a small time step must be used, it does not require a predictor-corrector method. When applying the Dufort–Frankel scheme, the absence of the $H_{i,k}$ value in the difference equation suggests that β is not related to $H_{i,k}$. On the other hand, using the average of $H_{i,k+1}$ and $H_{i,k-1}$ would make the finite-difference equations nonlinear. The average

$$\tfrac{1}{2}(H_{i+1,k} + H_{i-1,k}) \tag{33}$$

is therefore employed [8].

6. Comparative Analysis for a Monovalued Magnetic Relationship

Starting with the nonlinear problem, we will first consider a monovalued nonlinear relationship, and then introduce the bivalued function.

Given the linear character of the difference equation and in spite of the nonlinearity of the differential equation, the implicit schemes remain stable. Douglas and Jones [13] have shown that this fact is true when the Crank–Nicolson scheme, incorporating a predictor-corrector method, is applied to quite a comprehensive class of partial differential equations which includes equation (2). It is important to note that, given the form of the magnetic characteristic (Fig. 2b), the value of β is always positive; consequently the approximation is of a positive type, the stability being evident [2].

However, this question is not so clear for the explicit methods. It can be shown [10] that for the simple explicit scheme

$$r \leqslant \frac{1}{2}\left(1 - \Delta t \left|\frac{1}{\beta}\frac{\partial \beta}{\partial H}\frac{\partial H}{\partial t}\right|_{\max}\right) \tag{34}$$

is a sufficient condition for stability and convergence. However, Lim and Hammond [3] have found experimentally that Eq. (23) is satisfactory.

For the Dufort–Frankel scheme, Wiak [14] presents the conditions to solve the nonlinear monovalued problem although incorporating the Saulev difference diagram in the first row [15]. He proves the stability of the difference diagram by means of the energy method [15] and the conditions for the minimum error of the approximation.

With respect to convergence, the truncation errors maintain their structure. However, the strong nonlinear character of the B/H relation makes the field variation within the material faster than the linear one. Thus, for the difference equation to follow these variations, the space-time grid must be finer. This signifies that, to obtain a similar accuracy to the linear case, the size of the intervals must be smaller.

To summarize, the Dufort–Frankel scheme retains the essential advantage of an explicit method, but also retains the condition (30). The simple implicit and the Crank–Nicolson schemes are unconditionally stable for all the problems and do not suffer from any restriction in the relation $\Delta t/\Delta x$. However, the incorporation of a predictor-corrector method makes these implicit schemes slower. This fact, reduces the advantages of the Crank–Nicolson scheme with respect to the Dufort–Frankel scheme, which can be more suitable.

Figures 6, 7, and 8 show the above-mentioned considerations. We have represen-

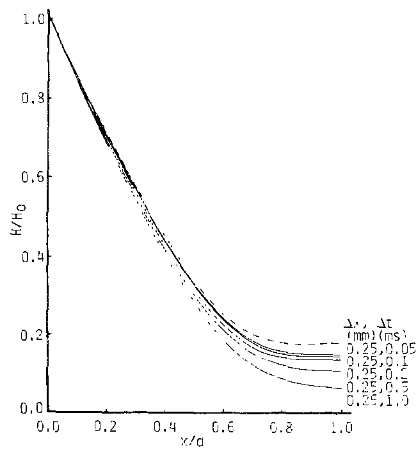

FIG. 6. Calculated amplitudes of H as a function of the depth in a nonlinear material with a monovalued B/H relationship, using the simple implicit scheme.

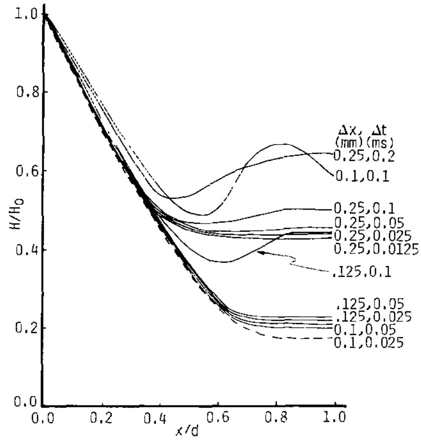

FIG. 7. Calculated amplitudes of H as a function of the depth in a nonlinear material with a bivalued B/H relationship, using the Dufort–Frankel scheme.

ted the calculated solutions obtained by means of the different schemes, taking as the magnetic characteristic the magnetization curve of the material considered. In this case, on the contrary to the linear problem, we do not know the exact solution, and so we have taken, as an approximation to this, the average of the optimal solutions of the two schemes with minimum error. The justification for this approximation is based on the fact that, in the linear problem, the Dufort–Frankel scheme converges to the theoretical solution by excess, while the Crank–Nicolson scheme converges by defect. From Figs. 7 and 8 this behavior appears to be the same in the nonlinear problem. This implies that we suppose both difference

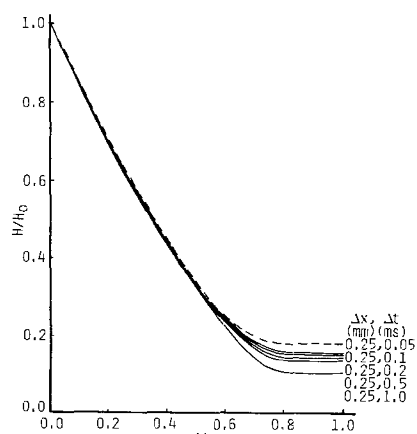

FIG. 8. Calculated amplitudes of H as a function of the depth in a nonlinear material with a bivalued B/H relationship, using the Crank–Nicolson scheme.

equations to converge to the same solution. This occurs if the same magnetic characteristic is considered independently of the fact that the difference equation correctly describes the physical phenomenon. The diffusion equation is solved for a steel plate with a half thickness $d = 2.5 \times 10^{-3}$ m, conductivity $\sigma = 5 \times 10^{6} \Omega^{-1}$ m^{-1} and with a magnetization curve that can be approximated by the Frolich curve: $B = H/(156 + 0.59H)$. The applied field is supposed to be 6000 A/m with a frequency of 50 Hz.

In the figures we can observe that the implicit schemes continue converging monotonously and quickly, only by reducing the time intervals, although a little slower than in the linear case. As before, the Dufort–Frankel scheme keeps to the convergence limitation, and if Eq. (30) is not fulfilled, the solution converges to a different solution of the diffusion equation.

7. Comparative Analysis for a Bivalued Magnetic Relationship

The introduction of a bivalued B/H relationship does not change the linear character of the difference equations. However, considering a hysteresis loop as the B/H relationship (Fig. 2c) implies that the $\partial B/\partial H$ function stays positive, but not continuous. Consequently neither is the coefficient of the difference equation and thus, it is more complex to prove the stability and convergence of the schemes [2]. It should be noted that the discontinuity is of jump type, always remaining finite and bounded by the value $(\partial B/\partial H)_{\max}$.

We have verified experimentally that the Dufort–Frankel scheme is not convergent with this B/H relationship, coinciding with the remarks made by other authors [3, 16].

Likewise, we have verified that the implicit schemes remain stable [17]. Given that the implicit scheme has the same disadvantages as the Crank–Nicolson scheme but has not the advantage of the large time step for a given accuracy, we can conclude that, of the schemes considered, the most adequate to solve numerically, the diffusion equation with a bivalued B/H relation, is the Crank–Nicolson scheme.

In Fig. 9 we can see the behavior of the solution obtained with this scheme for a sample of conductivity $\sigma = 4.21 \times 10^{-6} \Omega^{-1}$ m^{-1} and halfthickness $d = 7.5$ mm,. The magnetic characteristic is approximated by a series in arctan powers [17]. The applied field is 1000 A/m and the frequency 50 Hz. From Fig. 9 the good behavior of the solution can be verified, showing a great regularity in the convergence. In this case we cannot estimate a possible limit solution.

8. Bivalued Magnetic Relationship Variable with Depth

Given the instability of the Dufort–Frankel scheme, and the choice, among the implicit methods, of the Crank–Nicolson as the most adequate scheme to solve the

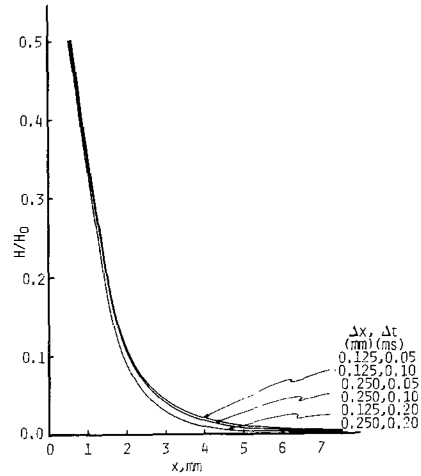

FIG. 9. Calculated amplitudes of H as a function of the depth in a nonlinear material with a bivalued B/H relationship, using the Crank–Nicolson scheme.

diffusion equation, we could state that the comparative analysis of the considered schemes is concluded. However, we must verify that this scheme remains suitable when a bivalued magnetic characteristic variable with depth is used. This type of characteristic is our main objective.

Such a magnetic characteristic supposes the existence of a different hysteresis loop each depth within the magnetic material, depending on the maximum amplitude reached by the field H. If the choice of the correspondent loop is not adequate, the difference equation cannot be compatible with the diffusion equation. The reason being as follows: Given that the choice of the loop depends on the maximum value reached by H, if this is evaluated by excess the magnetic characteristic followed will be a larger loop than the correct one, and thus, the $\partial B/\partial H$ factor will be evaluated by excess too. As this term is related to the field penetration in the material, the following maximum will be evaluated by defect, and hence the corresponding hysteresis loop will be smaller and in consequence the subsequent calculated maximum will be greater. This process can continue in such a manner that the solution obtained be a nonsymmetrical wave that practically never reaches the steady state conditions.

We have solved this problem by incorporating the accelerator procedure not only to the $H_{i,k}$ values, but also to the maximum values, because these determine the loop for each depth. It must be pointed out that the justification of this method is based on the application of a further boundary condition, extended now to the maximum values. If we call the maximum value reached by the calculated field on a depth $i \cdot \Delta x$ during the half cycle n

$$\bar{H}_{i,n} = (H_{i,k})_{\max}; \qquad k = (n-1)\,N/2,\, nN/2 \tag{35}$$

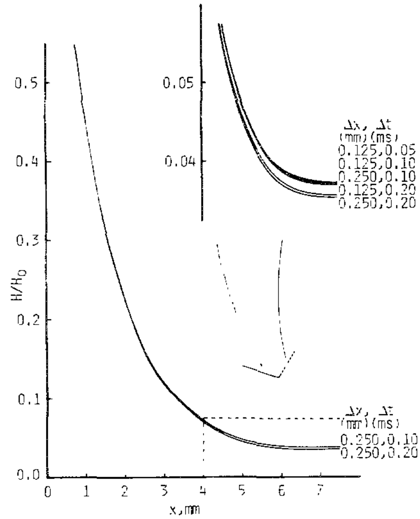

FIG. 10. Calculated amplitudes of H as a function of the depth in a nonlinear material with a family of bivalued, B/H relationship curves.

we must apply the corrector factor

$$\bar{K}_i = \tfrac{1}{2}(\bar{H}_{i,n} + \bar{H}_{i,n-1}) \tag{36}$$

and thus the corrected maximum values are

$$\bar{H}'_{i,n} = \bar{H}_{i,n} - \bar{K}_i = \tfrac{1}{2}(\bar{H}_{i,n} - \bar{H}_{i,n-1}). \tag{37}$$

With the application of this equation once every half cycle the steady state is reached with a few half-cycles, typically 5 to 10, depending on the saturation degree of the sample. Thus, the waves for each depth are symmetrical, reaching correctly the steady state.

In Fig. 10 we have represented the amplitude of the calculated solution using the Crank–Nicolson scheme and for the material of the Fig. 9. Here the magnetic characteristic is simulated by a family of static loops (see Fig. 11) approximated by

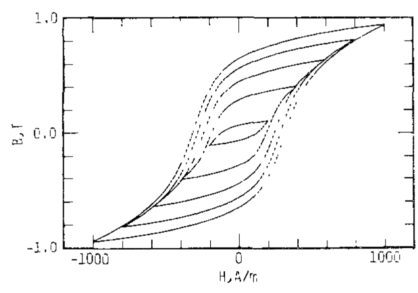

FIG. 11. Family of static loops used as magnetic characteristic.

a mathematical model [18] which allows us to know each of the interior loops solely from a representation of the exterior loop, the magnetization curve and the maximum value reached by H. The fast convergence of the solution can be seen. This is due to the application of the accelerator procedure to the maximum values reached by the field H, which determine the interior loop and guide the solution in such a manner that the convergence is quicker.

9. CONCLUSIONS

To select a finite difference scheme to solve the nonlinear diffusion equation, a comparative analysis of the main difference schemes was carried out. Given the difficulties introduced by the nonlinear magnetic relationship in the diffusion equation we have put forward an ordered comparative study. First, the linear problem was studied and thereby, knowing the solution, the references to make the comparison were fixed. Next, a nonlinear monovalued B/H relationship was considered and it was found that the Dufort–Frankel and the Crank–Nicolson schemes were the best. The former, being an explicit scheme and having a truncation error which can be similar to the Crank–Nicolson one, seems to be more suitable. However, the introduction of a nonlinear bivalued B/H relationship in the diffusion equation makes the Dufort–Frankel scheme unstable and thus, the Crank–Nicolson scheme gives the best results. It was verified that this scheme preserves its characteristics when the bivalued nonlinear B/H relationship depends on the depth in the sample. In conclusion the Crank–Nicolson scheme was selected to solve the nonlinear diffusion equation.

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